flame measurements demonstrated survivability and measurement capability in high heat flux environments.

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Thermally Developing Convection from Newtonian Flow in Rectangular Ducts with Uniform Heating

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Introduction

THE fluid flow and heat transfer behavior of laminar flow through rectangular ducts is of special interest because of the wide application of such a geometry in compact heat exchangers. Consequently, extensive studies have been carried out on such a geometry. An excellent comprehensive review of laminar flow forced convection in duct was presented by Shah and London.[1] Recently, Hartnett and Kostic[2] gave an up-to-date collection of literature in the heat transfer aspect for rectangular duct flows. The boundary conditions referred to by these authors fall into the three categories: 1) constant wall temperature both peripherally and axially, known as T boundary condition; 2) constant heat input per unit axial distance and constant peripheral wall temperature at each axial position with wall temperature varying axially only, referred to as $H1$ boundary condition; and 3) constant heat input per unit axial distance as well as per unit peripheral distance, this is denoted by $H2$ boundary condition.

Clark and Kays[3] and Schmidt and Newell[4] numerically evaluated the Nusselt number for rectangular ducts under both T and $H1$ boundary conditions. Sparrow and Siegel[5] developed a variational approach for heat transfer in rectangular ducts for the $H2$ boundary condition. All these studies are limited to the hydrodynamically and thermally fully developed condition.

A few investigators considered the case of hydrodynamically fully developed and thermally developing Newtonian flow. Montgomery and Wibulswas[6] used a finite-difference method to solve for the Nusselt number for rectangular duct for T and $H1$ boundary conditions. Chandrupatla and Sastri[7] obtained the developing Nusselt numbers under the T, $H1$, and $H2$ boundary conditions, but only for the square duct geometry.

Our literature survey reveals that there is a lack of general information and treatment on thermally developing laminar flow in rectangular duct for the $H2$ boundary condition. Complementary to previous studies, this work is to investigate the behavior of hydrodynamically fully developed and thermally developing laminar flow of Newtonian fluids in rectangular ducts for the $H2$ boundary condition.

Analysis

Consideration is given to the system in which a Newtonian fluid flows in a horizontal rectangular channel with an arbitrary aspect ratio subject to $H2$ boundary condition. Let the origin of the coordinates be set at a lower corner of the duct; the axial direction along which the fluid flows coincides with the z axis and the cross section area of the duct, $2a \times 2b$ is perpendicular to the z axis. The present analysis is based on the following assumptions: 1) hydrodynamically fully developed laminar flow of Newtonian fluids, 2) steady-state, 3) constant fluid properties, and 4) neglecting axial conduction and viscous dissipation.

The momentum equation of Newtonian fluid takes the form of

$$\frac{\partial^2 w}{\partial x^2} + \frac{\partial^2 w}{\partial y^2} = \frac{1}{\mu}\frac{\mathrm{d}p}{\mathrm{d}z} = \text{const} \tag{1}$$

Equation (1) is solved analytically by the separation of variables. The final solution is

$$w = \frac{g_c}{2\mu}\frac{\mathrm{d}p}{\mathrm{d}z}a^2\left\{\frac{32}{\pi^3}\sum_{n=1,3,5,\ldots}^{\infty}\frac{1}{n^3}(-1)^{(n-1)/2} \cdot\cos\left[\frac{n\pi(x-a)}{2a}\right]\frac{\cosh(n\pi(y-b)/2a]}{\cosh(n\pi b/2a)} - \left[1-\left(\frac{x-a}{a}\right)^2\right]\right\} \tag{2}$$

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The mean velocity is found to be

$$w_m = \frac{a^2}{3}\frac{g_c}{\mu}\frac{\mathrm{d}p}{\mathrm{d}z}\left[1 - \frac{192}{\pi^5}\frac{1}{\alpha^*}\sum_{n=1,3,5,\ldots}^{\infty}\frac{1}{n^5}\tanh\left(\frac{n\pi\alpha^*}{2}\right)\right] \tag{3}$$

where α^* is duct aspect ratio b/a. Based on the assumptions mentioned earlier, the dimensionless energy equation is written as

$$W(\partial\theta/\partial Z) = \partial^2\theta/\partial X^2 + \partial^2\theta/\partial Y^2 \tag{4}$$

where $X = x/D_h$, $Y = y/D_h$, $Z = z/(D_h RePr)$, $W = w/w_m$, D_h is hydraulic diameter, $\theta = (T - T_{in})/(q_0 D_h k)$, T_{in} is the temperature at the duct inlet, q_0 is the surface heat flux per unit area. The corresponding boundary conditions are: $\partial\theta/\partial X = -1$ @ $X = 0$, $Y \geq 0$, $Z \geq 0$; $\partial\theta/\partial X = 1$ @ $X = (1 + \alpha^*)/2\alpha^*$, $Y \geq 0$, $Z \geq 0$; $\partial\theta/\partial Y = -1$, @ $X \geq 0$, $Y = 0$, $Z \geq 0$; $\partial\theta/\partial Y = 1$, @ $X \geq 0$, $Y = (1 + \alpha^*)/2$, $Z \geq 0$ and $\theta = 0$ @ $X \geq 0$, $Y \geq 0$, $Z = 0$.

A finite-difference scheme is developed to solve the system mentioned above. The backward difference and five point central difference are employed for the convection term and the conduction term, respectively. The temperature at a given position along the duct is obtained from the following implicit form:

$$\begin{aligned}\overset{(I+1)}{\theta}(L, M) = {} & \omega(R_1\{16[\overset{(I)}{\theta}(L + 1, M) + \overset{(I+1)}{\theta}(L - 1, M)] \\ & - \overset{(I)}{\theta}(L + 2, M) - \overset{(I+1)}{\theta}(L - 2, M)\} + R_2\{16 \\ & \cdot[\overset{(I)}{\theta}(L, M + 1) + \overset{(I+1)}{\theta}(L, M - 1)] - \overset{(I)}{\theta}(L, M + 2) \\ & - \overset{(I+1)}{\theta}(L, M - 2)\} - d_2\overset{(I)}{\theta}(L, M) + d_1)/d_2 + \overset{(I)}{\theta}(L, M)\end{aligned} \tag{5}$$

where L and M are the nodal points in X and Y directions, respectively, $\theta(L, M)$ represents the temperature at the position $X = (L - 1)DX$, $Y = (M - 1)DY$, and $Z = (J + 1)DZ$; DX, DY, and DZ are dimensionless increments in X, Y, and Z directions, respectively; I represents the iteration counter; ω is the overrelaxation factor being 1.3–1.5 in the present calculations.

$R_1 = DZ/[12(DX)^2]$, $R_2 = DZ/[12(DY)^2]$, $d_1 = W(L, M)\theta_1(L, M)$, $d_2 = W(L, M) + 30R_1 + 30R_2$, where $\theta_1(L, M)$ is the temperature of the previous duct cross section at $Z = (J)DZ$. The temperature distribution at Jth station along the axial direction is employed as an initial guess for the temperature at $(J + 1)$th station. The imaginary values of temperature exterior to the duct are required when solving Eq. (5). They are obtained by a five point extrapolation formula. Once the temperature distribution at the internal nodes is obtained using the successive overrelaxation method, the temperatures at the boundaries are found using forward or backward five point central difference formula. With the temperature distribution at hand, the mean temperature across the duct section is calculated from

$$\theta_m = \frac{4\alpha^*}{(1 + \alpha^*)^2}\int_0^{(1+\alpha^*)/2}\int_0^{(1+\alpha^*)/2\alpha^*} W\theta \,\mathrm{d}X\,\mathrm{d}Y \tag{6}$$

The average Nusselt number of individual wall at a given duct section is expressed as

$$Nu_w = 1\bigg/\left[(D_h/L_i)\int_0^{L_i/D_h}\theta_w \,\mathrm{d}(\zeta/D_h)\right] \tag{7}$$

where L_i is the length of individual wall, and θ_w is the wall temperature. The average Nusselt number at any position Z is

$$Nu(Z) = 1/(\theta_{wm} - \theta_m) \tag{8}$$

where the mean temperature of the duct wall is defined by

$$\theta_{wm} = \frac{1}{s}\int_s \theta_w \,\mathrm{d}s \tag{9}$$

and s is the perimeter of the duct.

Results and Discussion

The input values of the present computations are listed in Table 1. The accuracy of the numerical results is of fourth order and first order in DX (or DY) and DZ, respectively. Iteration is considered to converge if the temperature difference between the two successive iterations agrees to a chosen value, 10^{-6}.

Figure 1 illustrates the developing Nusselt numbers in a rectangular duct for $H2$ boundary condition with the aspect ratio ranging from 1/8 to 1.0. In general, the developing Nusselt number increases as α^* decreases. The present limiting solutions of the fully developed Nusselt number agree excellently with those of Shah and London.[1] For example, for aspect ratio $\alpha^* = 1.0$, the present solution of fully developed Nusselt number is 3.099, while their solution is 3.091. Figure 2 shows the Nusselt number as a function of Z for $\alpha^* = 0.5$. Because of the symmetry in geometry and boundary conditions, the Nusselt numbers on each pair of opposite walls are

Table 1 Number of nodes for duct cross section and mesh size along axial direction employed in the present computations

α^*	NX	NY	DZ
1.0	21	21	0.00025
$\frac{1}{2}$	41	21	0.00025
$\frac{1}{3}$	61	21	0.00025
$\frac{1}{4}$	57	17	0.00025
$\frac{1}{5}$	75	15	0.00025
$\frac{1}{6}$	85	15	0.00025
$\frac{1}{8}$	97	13	0.00025
$\frac{1}{10}$	101	12	0.00025

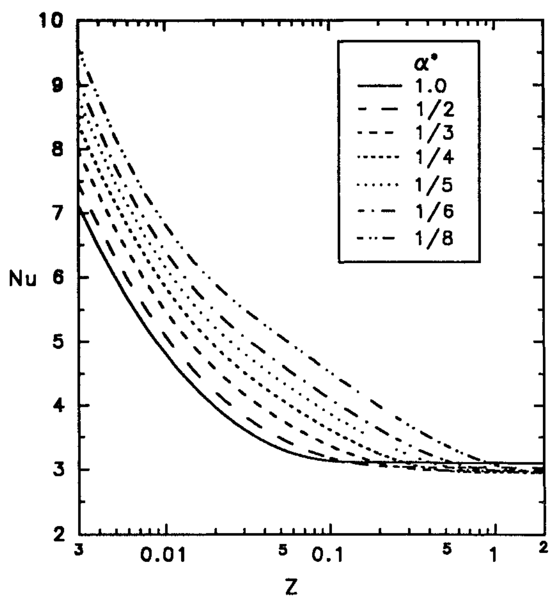

Fig. 1 Thermally developing Nusselt numbers for Newtonian fluids for $H2$ boundary condition with duct aspect ratios ranging from 1/8 to 1.0.

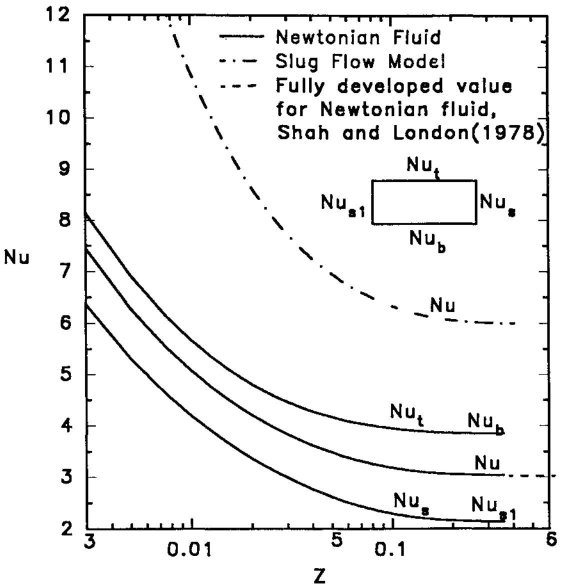

Fig. 2 Thermally developing Nusselt numbers as a function of Z for $H2$ boundary condition, with $\alpha^* = 1/2$.

Table 2 Comparison of thermal entrance lengths under various boundary conditions

	Present	Hartnett and Kostic (1989)	
α^*	L^*_{thH2}	L^*_{thH1}	L^*_{thT}
0.000	∞	0.0125	0.0080
0.100	1.312	0.0260	0.0422
0.125	0.883	0.0300	0.0450
0.167	0.550	0.0336	0.0500
0.200	0.403	0.0370	0.0521
0.250	0.290	0.0421	0.0541
0.333	0.175	0.0482	0.0539
0.400	0.135	0.0520	0.0520
0.500	0.104	0.0566	0.0490
0.750	0.077	0.0636	0.0446
1.000	0.065	0.0660	0.0410

identical, (i.e., $Nu_b = Nu_t$, $Nu_s = Nu_{s1}$). The corresponding slug flow model ($W = 1$) is included as a dash line in this figure. Although the slug flow solutions for various aspect ratios are not presented here, our computation reveals that the fully developed Nusselt number for slug flow approaches a value of 6.00, independent of the duct aspect ratio. This is in contrast to the case of Newtonian fluids for which the fully developed Nusselt number for square duct ($\alpha^* = 1$) appears to be the upper bound for all curves shown in Fig. 1.

The thermal entrance length which is defined as the duct length at which the local Nusselt number has reached 5% of the fully developed value, L^*_{th} is listed in Table 2 for some selected values of α^*. The comparison of L^*_{th} is made under aforementioned boundary conditions. Unlike the case of T and $H1$ boundary conditions reported by Hartnett and Kostic,[2] the thermal entrance length for $H2$ boundary condition does not approach the same value for plane parallel plate as $\alpha^* \rightarrow 0$. Instead, L^*_{thH2} goes to infinity when α^* approaches zero.

Conclusion

The forced convective heat transfer for the laminar flow under the $H2$ boundary condition in rectangular ducts is studied numerically. The developing Nusselt number is obtained for a wide range of duct aspect ratios. The limiting solution of Nu_{H2} as $Z \rightarrow \infty$ agrees excellently with those found in the literature. The thermal entrance length for this case shows a totally different trend from the counterpart of T and $H1$ boundary conditions. It increases monotonically as α^* decreases from one to zero.

The present analysis and discussion are restricted to the case of equal heating on adjacent walls. The analysis can be easily extended to the case of unequal heatings on adjacent walls. This latter case has significant applications in electronic industry. Some numerical examples for unequal heatings on adjacent walls for rectangular ducts were illustrated by these authors, see Chung et al.[8]

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Approximation Method for Rate of Appearance of Temperature Distributions in Spherical Objects

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Introduction

HEAT flow equations are the basis of many engineering applications, but the complexity of the equations makes it difficult to estimate temperature distributions, even in a semiquantitative manner. It would therefore be useful to have a method to approximate the rate at which the temperature distributions develop during the course of heat treatment.

Model for Spherical Objects

For the purposes of this model, the object will be assumed to have spherical symmetry and uniform composition, where

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